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RESOURCE ALLOCATION IN MULTI-LEVEL SPATIAL HEALTH CARE SYSTEMS: BENEFIT MAXIMISATION.

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1. INTRODUCTION

This paper introduces the theory and first results of a set of possible methods for allocating resources in a region to different services or sectors of a health care system. This problem is one typically faced by providers in countries where some planning controls exist (see, DHSS, 1976 for examples) and where there is a desire to develop a balanced pattern of provision. The resultant models arising from the theory may be regarded as a first step towards a generalisation of the efficiency principle given in Mayhew, Leonardi (1981).

This principle is designed to pick allocations that maximise the benefits to patients given their preferences for treatment in different locations. Work is also in progress aimed at extending the approach to the equity criterion, also considered in this reference, and this should make interesting comparison. The main aim of the current paper is to give a shortened introduction to the models, using illustrations from two allocation sub-problems, and then to present some preliminary results based on health care services in the state of Massachusetts, USA.

2. BACKGROUND

In a health care system, resources are often scarce, with the result that there is considerable pressure on available beds, doctors, nurses, and support services. The dilemma faced by providers is that, despite substantially increased budgets for health care during the last 30 years, this pressure has not slackened. Priorities over resources arise for medical reasons, teaching obligations, research needs, or because a government or health authority wants to devote special attention to certain services (LHPC, 1979). However, if the population structure in a region is changing, and when there are other factors influencing the uptake in services, then there is a high chance of the resultant allocation process becoming haphazard.

In an ideal world, it would be correct to base allocations on strictly defined medical grounds. However, such is the breadth of services covered by a health care system, and because the outcomes and benefits of so many medical procedures involved are so hard to evaluate, this ideal is simply impractical as yet. The approach considered here is rather more pragmatic; it argues that allocations are part of a behavioural process in which the principle driving force among the various actors is to satisfy potential demand. It recognises, however, that there are many constraints, side objectives, and pressures in the system that prevent this objective being met. In the theory, the idea is that these aspects are taken care of by two sets of factors called thresholds and bounds.

Thresholds are based on the minimum acceptable level of potential demand before a service can be provided in a particular treatment district. They can be regarded as service norms or certificates of need laid down by health ministries, authorities, agencies, or medical opinion. Alternatively, they may arise from economic considerations internal to the system, such as economies of scale, or they may reflect certain licencing laws if there are legal restrictions on the provision of some services. For those services involving specialised and expensive therapeutics, thresholds will be high relative to potential demand; for other services they may be nonexistent, in which case provision will be completely routine.

Bounds, by contrast, are constraints on the *total* allocations to any destinations. They represent a variety of considerations such as physical restrictions on facility expansion and the availability of finance capital. They can also be influenced by political considerations, teaching and training needs, community pressure, and other factors, preventing the run-down or closure of facilities that might otherwise be expected to take place. What the model does is simply to choose a path through both sets of factors - the thresholds and bounds - consistent with the benefit maximisation principle (and, in future work, the equity principle also).

The main outputs from the model will be a set of caseload allocations to destinations by service category.

The results look like a service hierarchy that, in theory, is responsive to budget levels, the changing spatial configuration of potential demand, accessibility costs, allocative priorities and what is feasible in the system to change.

3. THE MODELS

In the following sectors an overview of the models and methods will be given. They build on the approach developed in Mayhew, Leonardi (1981), which in turn is based on related work developed at the International Institute for Applied Systems Analysis (IIASA), Austria, in the Health Care and Public Facility Location Tasks [also in conjunction with the Operational Research Services of the Department of Health and Social Secutiry (DHSS), UK]. Published work in the field includes Gibbs (1978); Hughes, Wierzbicki (1980); Aspden (1980); Aspden, Gibbs, Bowen (1980); Aspden, Rusnak (1980); Mayhew (1980), Mayhew, Taket (1980 b); Rousseau, Gibbs (1980); Mayhew (1981), Mayhew, Taket (1981), but the general philosophy goes back further to McDonald, Cuddeford, Beale (1974). One of the spatial allocation subproblems considered, however, embeds an attraction constrained gravity model, which is well known to regional scientists and hence draws on a separate tradition (e.g., Wilson 1974, Batty 1976, Leonardi 1978, 1980a, 1980b, 1981).

In a health context, the latter model would presume that all resources at facilities in a destination zone are used to capacity. Though, as will be seen, the reality is slightly more complex this assumption is largely in harmony with the observed behaviour of the system, particularly the apparent incessant pressure on services and the way in which demand seems to rise so that is meets supply (Feldstein, 1963; RAWP, 1976; Gibbs, 1978; Mayhew, Taket, 1980a). In the following sections an overview of part of the efficiency criterion will be given, but further results, including those for the problem with bounds, and also a description of calibration methods, algorithms, and possible implementation procedures, will be presented at a later time. The first step is to define the full problem and the utility function, which links the methods together, and then to break the problem down into three simpler sub-problems. First results are then given.

4. THE PROBLEM

The full problem may be stated as follows:

$$\max_{D_{jk}} \sum_{jk} \delta_{jk} \, g_k \, (D_{jk}) \qquad (1)$$

subject to

$$\sum_{jk} D_{jk} \delta_{jk} = Q \qquad (2)$$

$$R_j \leq \sum_k D_{jk} \delta_{jk} \leq S_j \qquad (3)$$

$$D_{jk} \geq A_k \quad \text{if } \delta_{jk} = 1 . \qquad (4)$$

Equation (1) is the function to be maximised, where $g_k(D_{jk})$ is a utility function (see Section 4.1), D_{jk} are the (as yet) unknown resources (measured in caseloads) allocated to destination j in service category k, and where δ_{jk} is a binary matrix in which elements are set to 1 if a service k is provided in j and zero otherwise.

Equation (2) is a budget constraint based on the total treating capacity of the system. Condition (3) represents the upper and lower bounds on the allocations to a destination (S_j and R_j, respectively). Condition (4) is the threshold principle in which the threshold in service sector k is given as A_k. Note also that the objective function is presumed to operate over the whole system, implying the existence of a high level decision making authority, but where local conditions are taken care of in (3). When bounds are tight, either it implies a lot of friction in the system (due to lack of finance, say) or it suggests a high degree of local autonomy. In both cases it would mean less room for manoever at the higher level. In Section 4.3, we contrast the utility at this higher level utility, which is based on which particular patients to treat.

4.1. The Utility Function

The utility function used in this version of the model has the same mathematical form as the one described in Aspden (1980), given by:

$$g_k\ (D_{jk}) = \frac{\Phi_{jk}}{\alpha_k}\left[1 - \left(\frac{D_{jk}}{\Phi_{jk}}\right)^{-\alpha_k}\right] \qquad (5)$$

where

Φ_{jk} is a non-negative quantity proportional to the ideal level of demand for service category k in location j; from now on it will be called the <u>potential demand,</u>

α_k is a positive parameter reflecting the priority the system gives to service category k.

The function defined in (5) is clearly increasing with respect to the allocation variable D_{jk}. As for its behavior with respect to Φ_{jk}, it is easily shown by elementary calculus that it is increasing when Φ_{jk} is within the interval:

$$0 \leq \Phi_{jk} \leq D_{jk} \left(\frac{1}{1 + \alpha_k} \right)^{1/\alpha_k}$$

while it is decreasing outside this interval. Although the existence of a decreasing portion of the utility function makes no sense intuitively, its disturbing effect can be easily eliminated in practice. Again elementary calculus yelds the inequality:

$$e^{-1} \leq \left(\frac{1}{1 + \alpha_k} \right)^{\alpha_k} \leq 1$$

therefore a sufficient condition for the utility function being increasing with respect to Φ_k is:

$$\Phi_{jk} \leq e^{-1} D_{jk} \cong 0.37 D_{jk} .$$

The optimization problems to be considered in this paper can be always formulated in such a way as to meet the above inequality in the meaningful range of feasible solutions. Indeed, constraint (4) is enough to conclude that, if D_{jk} is positive, it is larger than a given threshold A_k, which is typically a large number (several thousands). On the other hand, the quantities Φ_{jk} are defined up to a multiplicative constant, so that they can be arbitrarily rescaled in order to be made less than unity, say (or any other positive number one wishes), hence the above statement follows.

Later sections of the paper will be more specific an assumptions concerning Φ_{jk}. Here it suffices to say that in general:

$$\Phi_{jk} = f\,(W_{jk}, \omega_{jk}, C_{ij}, \beta_k).$$

That is, Φ_{jk} is a function of the patient generating factor in place of residence i, category k; a factor ω_{jk}, presumed constant, related to the importance of satisfying potential demand and the prestige of the facilities in j offering k; accessibility costs C_{ij}; and a space discount parameter β_k. In Section 4.3, the link is made with the gravity model, at which time Φ_{jk} will be specified in more detail.

The parameter α_k, meanwhile, reflects the priority given to services at different budget levels, Q. A service is said to be inelastic (high α), for example, if treatment cannot easily be deferred without causing physical distress and medical complications. In this case, caseloads change proportionately very little. By contrast, services that are elastic (low α), because treatment can be deferred, respond proportionately more when budgets rise or fall.

Before proceding, it should be noted that the approach ignores possible variations in treatment *standards*, which are also elastic to different budget levels (Gibbs, 1978). Though it is possible to allow for this in the methods, it is presumed for current purposes that the subset of services being examined have relatively constant treatment requirements. This assumption would be met if the services being considered were in certain acute categories.

4.2. Spatial and Sectoral Allocations Without Constraints

If there are no bounds, the sectoral allocation problem separates from the spatial problem. Further, if there are no bounds *and* no thresholds then sectoral allocations are consistent with potential demand ϕ and medical priorities. First, define the following

$$D_k = \sum_{j \in L_k} D_{jk} \,. \tag{6}$$

The resources allocated to sector k, and p_{jk}, the share of resources for k allocated to zone j, such that

$$\sum_{j \in L_k} p_{jk} = 1 \,. \tag{7}$$

In (6) and (7) L_k is the set of destinations with k allocated (i.e., $\delta_{jk} = 1$) and D and p are related by

$$D_{jk} = D_k \, p_{jk} \,. \tag{8}$$

4.2.1. Spatial Allocation

Substituting (8) into (5) one gets:

$$\bar{g}_k\,(D_k, p_{jk}) = g_k\,(D_k\,p_{jk}) =$$

$$= \frac{\phi_{jk}}{\alpha_k}\left[1 - \left(\frac{D_k\;p_{jk}}{\phi_{jk}}\right)^{-\alpha_k}\right].$$

When the sectoral allocation D_k is held constant, the spatial allocation is obtained by solving for each k the following optimization problems:

$$\max_{p} \sum_{j \in L_k} \bar{g}_k\,(D_k, p_{jk}) \tag{9}$$

subject to constraint (7).

Although it is not strictly needed now, it is also useful to keep in mind the constraint on sectoral allocation

$$\sum_k D_k = Q\;. \tag{10}$$

The spatial allocation solution is given by the optimality conditions

$$\frac{\partial}{\partial p_{jk}}\,\bar{g}_k\,(D_k, p_{jk}) = \lambda_k \tag{11}$$

where λ_k is the multiplier for (7). That is,

$$D_k \left(\frac{D_k \, p_{jk}}{\phi_{jk}} \right)^{-(1+\alpha_k)} = \lambda_k \; . \qquad (12)$$

Defining

$$\Psi_k = \frac{1}{D_k} \left(\frac{\lambda_k}{D_k} \right)^{-1/(1+\alpha_k)} \qquad (13)$$

we have

$$p_{jk} = \Psi_k \, \phi_{jk} \; . \qquad (14)$$

From constraint (7), however,

$$1 = \Psi_k \sum_{j \in L_k} \phi_{jk} \; . \qquad (15)$$

Letting

$$\phi_k = \sum_{j \in L_k} \phi_{jk} \; . \qquad (16)$$

then from (14) and (15) it is seen that

$$p_{jk} = \frac{\phi_{jk}}{\phi_k} \; . \qquad (17)$$

Thus, the spatial allocation of resources is independent from the sectoral allocation. It depends on the potential demand incident on j in k and on the total potential demand in all j. This result is hence a generalisation of the efficiency principle to many services (Mayhew, Leonardi, 1981).

4.2.2. Sectoral Allocation

Sectoral allocations are those to each k. Substitute (17) in (1) and consider the following problem

$$\max_{D_k} \sum_k \sum_{j \in L_k} \bar{g}\left(D_k, \frac{\phi_{jk}}{\phi_k}\right) \tag{18}$$

subject to constraint (10). The optimality conditions are

$$\left(\frac{D_k}{\phi_k}\right)^{-(1+\alpha_k)} = \lambda \tag{19}$$

where λ is the multiplier for (10). Hence,

$$D_k = \phi_k \lambda^{-1/(1+\alpha_k)}. \tag{20}$$

For a given L_k, λ is found as the root of the equation

$$Q = \sum_k \phi_k \lambda^{-1/(1+\alpha_k)} \tag{21}$$

which can be found by using the Newton-Raphson method.

4.2.3. *Allocations with Thresholds*

When a particular allocation does not meet a threshold, then resources must be reallocated. This process is called allocation by forced substitution. From (17) and from the definition of p_{jk}, we have

$$D_{jk} = D_k \frac{\phi_{jk}}{\phi_k} \tag{22}$$

To cross the threshold A_k, therefore,

$$D_k \frac{\phi_{jk}}{\phi_k} \geq A_k \tag{23}$$

or

$$D_k \geq \frac{A_k \phi_k}{\phi_{jk}} \quad \forall \text{ j and k} \tag{24}$$

otherwise there are forced substitutions. The optimality conditions for this problem are, from (10), (15), and(24)

$$\left(\frac{D_k}{\phi_k}\right)^{-(1+\alpha_k)} - \lambda + \xi_k = 0 \tag{25}$$

where ξ_k are the multipliers for (24) and where $(\lambda - \xi_k) > 1$. ξ_k is active (i.e., non zero) when

$$D_k < \frac{A_k \phi_k}{\min\limits_{j \in L_k} \phi_{jk}} . \tag{26}$$

Combining (25) and (26) yields for forced substitutions

$$\lambda > \min_{j \in L_k} \left(\frac{\phi_{jk}}{A_k}\right)^{1+\alpha_k} . \tag{27}$$

Conversely, no forced substitutions occur whenever

$$\lambda \leq \left(\min_{j \in L_k} \frac{\phi_{jk}}{A_k}\right)^{1+\alpha_k} . \tag{28}$$

Although the details of the algorithm to solve the combinatorial problem of meeting thresholds will not be given, its main ideas will be outlined here.

First of all, it should be noted that, due to the may the utility function is built, an optimal location policy will always imply to open as many locations, as possible and with the highest possible potential.

Secondly, if one defines the coefficients:

$$C_{jk} = \left(\frac{\phi_{jk}}{A_k} \right)^{1+\alpha_k}$$

then condition (28) can be re-written as:

$$\lambda \leq \min_{j \in L_k} C_{jk} .$$

It follows that the set of optimal locations for service category k, L_k, must be of the form:

$$L_k = \{j : C_{jk} \geq \lambda\}$$

for some λ root of equation (21). Of course many pairs $\{(L_k), \lambda\}$ exist which are consistent with the above form (and the corresponding L_k are all <u>subsets</u> of the optimal ones), but only one of such pairs has the highest cardinality for the L_k (that is, the maximum number of open locations) and this is the optimal solution.

The algorithm which has been devised is an iterative procedure which produces increasingly tighter upper and lower bounds on λ and on the L_k, thus converging to the optimal pair.

In order to show how the algorithm works, the first two steps will be described.

Start with $L_k^0 = \{j : j = 1,\ldots, N\}$, that is with <u>all possible locations open</u>. This is clearly an <u>upper bound</u> on L_k, i.e.:

$$L_k \subseteq L_k^0$$

and also the root of equation (21), λ_0, is an upper bound on λ, i.e.:

$$\lambda \leq \lambda_0$$

To get a first lower bound, consider the sets:

$$L_k^1 = \{ j : C_{jk} \geq \lambda_0 \}$$

and find the corresponding root λ_1 of equation (21).

The following statement is straightforward:

$$L_k^1 \subseteq L_k \subseteq L_k^0 \ ,$$

$$\lambda_1 \leq \lambda \leq \lambda_0 \ .$$

Thus the first two steps of the iterations have produced upper and lower bounds for the optimal solution. For the second two steps, one finds the sets:

$$L_k^2 = \{ j : C_{jk} \geq \lambda_1 \}$$

the corresponding root of equation (21), λ_2, the sets:

$$L_k^3 = \{ j : C_{jk} \geq \lambda_2 \}$$

and the corresponding root of equation (21), λ_3. It can be easily shown that:

$$L_k^1 \subseteq L_k^3 \subseteq L_k \subseteq L_k^2 \subseteq L_k^0$$

$$\lambda_1 \leq \lambda_3 \leq \lambda \leq \lambda_2 \leq \lambda_0 \, .$$

Therefore the bounds on the estimate of the optimal solution have been tightened. Repeating the same procedure over and over, the tightest possible bounds on the solution are obtained in a finite number of steps. The algorithm will eventually lead to a single solution, which is of course the optimal one (this often happens in actual computational experience), or to a small subset of feasible solutions, on which refined search to detect the optimal one can be easily performed. It should be remarked, however, that this final step is often uninteresting for practical purposes, since the last lower bound produced by the iterations is usually undistinguishable from the "true" optimum, both in terms of the value of the objective function and in terms of the corresponding location pattern.

4.3. The Gravity Mechanism

The spatial mechanism is, as was seen in Section 4.1, implicit in the methods developed. So far, we have allocated resources to destinations and sectors, but not to places of residence. For this we need the gravity model, but to use standard gravity parameters in the previous methods (see condition 3, Section 4.1, β_k), we have to make the links explicit. In doing so, it is important to emphasize that, in contrast to the utility of the allocation decisions originating higher up in the system, the decision on *which* particular patients to admit are purely local ones. Thus, it is necessary to maximise the utility only of the treatment destination, once the resources it has to dispense to the surrounding population have been determined. Accordingly, we define the following new maximisation problem

$$\max_{T_{ijk}} - \sum_i \frac{\phi_{ijk}}{\alpha_k} \left(\frac{T_{ijk}}{\phi_{ijk}} \right)^{-\alpha_k} \tag{29}$$

subject to

$$\sum_i T_{ijk} = D_{jk} \tag{30}$$

where T_{ijk} is the number of patients from i treated in j, service category k, where ϕ_{ijk} ($\phi_{jk} = \sum_i \phi_{ijk}$) is the potential demand incident on j from i, and where (30) is the resource constraint on the destination arising from higher level allocation processes, the effects of the bounds and thresholds having already been taken into account.

This is hence something like before except that we are just summing over places of residence i and not over j and k. Note also that α_k does not take into consideration possible differences between each j. In a well organized system, with a free flow of information, medical priorities should be perceived in more or less the same way, independently of location, but some empirical work may be necessary to check this. Continuing with the maximisation problem we have the following optimality conditions

$$\lambda = \left(\frac{T_{ijk}}{\phi_{ijk}}\right)^{-(1+\alpha_k)} \tag{31}$$

so that

$$T_{ijk} = \phi_{ijk}\, \lambda^{-1/(1+\alpha_k)} \tag{32}$$

From (30) however,

$$D_{jk} = \lambda^{-1/(1+\alpha_k)} \sum_i \phi_{ijk}\,. \tag{33}$$

Hence

$$T_{ijk} = \frac{D_{jk}\,\phi_{ijk}}{\sum_i \phi_{ijk}}\,. \tag{34}$$

Letting,

$$\phi_{ijk} = \omega_{jk}\, W_{ik}\, e^{-\beta_k C_{ij}} \tag{35}$$

where the right-hand variables were defined in Section 4.1, and substituting in (34), we have on cancelling the ω_{jk}'s a standard attraction constrained gravity model

$$T_{ijk} = B_{jk} D_{jk} W_{ik} e^{-\beta_k C_{ij}} \tag{36}$$

where

$$B_{jk} = \left[\sum_i W_{ik} e^{-\beta_k C_{ij}} \right]^{-1} . \tag{37}$$

5. FIRST RESULTS FROM MASSACHUSETTS

Several applications of one or more of the above or related methods are in hand or underway. The results presented here are the first so far obtained using the threshold mechanism in section 4.2.3. The necessary data were obtained for a 28 x 23 origin-destination system, 23 destinations corresponding to the health planning sub-areas in the state of Massachusetts, USA*. Apart from pro

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* The data on patient discharges were obtained through the cooperation of the University of Massachusetts, Amherst, and the Western Massachusetts Health Planning Council. Preparation was carried out by Mr. Richard Segall, of the Department of Industrial Engineering and Operations Research at the University, and Brandon Delaney, Ph.D., Research Director of the Council.

viding the first opportunity to study the approach, the US health care system presents several distinct challenges to the regional scientist. With no central decision maker, corresponding to a strong regional health authority, there is a high degree of local autonomy, implying (in our context) tight lower bounds because hospitals do not very easily give up their "limited" resources. On the other side, the changing configuration of potential demand, community pressure groups, Health Service Agencies, and insurance companies act to try to counter-balance the possibility of misdirecting resources. Contrary to opinion, the latter are market mechanism only up to a point because of the many distortions that are inherent in any health care "market". Recently, Certificates of Need have become a feature of the US scene, and these represent a higher level of control than hitherto. Again, in the current context they might be said to represent thresholds.

The spatial parameter β_k was determined for four services shown in Table 1 using the model in equation (32), based on population, utilization rates, patient flow data and accessibility costs*. The results showed that the model hypothesis was appropriate for all the services considered. The parameters α_k and ω_{jk}, however, were not estimated at this stage, their values being guessed or inferred from previous studies. Because of this, the re-

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* Based on work carried out by Professor E. Rising at the International Institute for Applied Systems Analysis whilst he was on leave from the University of Massachusetts, Amherst, USA.

sults should be regarded simply as a "test" of mechanisms. Likewise, the thresholds were chosen according to what seemed plausible. For example, for obstetrics, we set a low threshold (relative to potential demand), but a high α implying a routinely provided service that is inelastic to budget changes. Conversely, we gave a higher threshold (relative to potential demand) to acute psychiatric services and a low α.

In the following illustrative outputs (Table 2) we consider a set of allocations without thresholds and a set with. To keep the comparison as simple as possible, the budget Q is held constant at a level corresponding to the allocations for these four services in Massachusetts in 1978. Also no change is made to the geographical configuration of demand, although this would be easy to do. The following points may be made.

1. Six services are withdrawn in category one, two in two, nine in three, and 18 in four. In two destinations all services are withdrawn.

2. For a fixed budget, closing departments releases resources for other destinations. Thus, elsewhere allocations increase, but in accordance with α. In the four places remaining open to psychiatric services, the local budget increases by 11.1 percent as compared with 4.6 percent for obstetrics services.

3. *Total* budgets for each service can rise or fall ac—

cording to interactions between thresholds, elasticities, and potential demand. Here only the fourth changes significantly.

4. Although we do not show this, the closure of services can cause large changes in hospitalisation rates in those services in spite of the overall budget being maintained; the resources are simply redistributed among other services and destinations. This seems intuitive from equation (36), and thus it should also be taken into account (in fact, the equity criterion under development would consider this).

Table 1. Parameter values used for each service.

Type	β	R^2	α	A
Medical/Surgical	0.23	0.95	3	5,000
Obstetrics	0.27	0.95	6	300
Paediatrics	0.21	0.92	2	1,000
Psychiatric	0.25	0.96	2	300

β = spatial discount parameter (with variance explained by gravity model).

α = elasticity parameter reflecting systems' priorities.

A = thresholds on services.

R^2 = coefficient of explained variance.

Table 2. Allocation and reallocation with thresholds.

Treatment district	$\alpha = 3, k = 1, A_1 = 5000$ (1)[a]	(2)[b]	Pc change	$\alpha = 6, k = 2, A_2 = 300$ (1)	(2)	Pc change	$\alpha = 2, k = 3, A_3 = 1000$ (1)	(2)	Pc Change	$\alpha = 2, k = 4, A_4 = 300$ (1)	(2)	Pc Change
1	7507.	0.	-100.0	1012.	1059.	4.6	777.	0.	-100.0	158.	0.	-100.0
2	5610.	0.	-100.0	626.	0.	-100.0	739.	0.	-100.0	110.	0.	-100.0
3	30926.	33466.	8.2	4565.	4776.	4.6	3549.	3943.	11.1	695.	0.	-100.0
4	13124.	0.	-100.0	2116.	2214.	4.6	1506.	0.	-100.0	303.	0.	-100.0
5	1655.	0.	-100.0	133.	0.	-100.0	224.	0.	-100.0	27.	0.	-100.0
6	8231.	0.	-100.0	1173.	1227.	4.6	1088.	0.	-100.0	175.	0.	-100.0
7	22166.	23987.	8.2	3769.	3943.	4.6	2944.	3270.	11.1	539.	0.	-100.0
8	24345.	26344.	8.2	3350.	3505.	4.6	2865.	3183.	11.1	545.	0.	-100.0
9	18050.	19533.	8.2	2240.	2343.	4.6	2169.	0.	-100.0	422.	0.	-100.0
10	25676.	27785.	8.2	2610.	2730.	4.6	3535.	3927.	11.1	477.	0.	-100.0
11	76806.	83114.	8.2	10872.	11374.	4.6	9439.	10486.	11.1	1694.	1881.	11.1
12	79856.	86414.	8.2	12799.	13390.	4.6	9719.	10797.	11.1	1856.	2062.	11.1
13	26873.	29080.	8.2	3658.	3827.	4.6	3672.	4080.	11.1	568.	0.	-100.0
14	45424.	49154.	8.2	6043.	6322.	4.6	5608.	6231.	11.1	952.	1058.	11.1
15	16527.	17884.	8.2	2823.	2954.	4.6	2056.	0.	-100.0	387.	0.	-100.0
16	24988.	27040.	8.2	4304.	4503.	4.6	3356.	3729.	11.1	577.	0.	-100.0
17	24542.	26557.	8.2	3048.	3188.	4.6	2552.	0.	-100.0	554.	0.	-100.0
18	8368.	0.	-100.0	862.	902.	4.6	717.	0.	-100.0	153.	0.	-100.0
19	24730.	26761.	8.2	3349.	3503.	4.6	2989.	3321.	11.1	523.	0.	-100.0
20	33542.	36297.	8.2	4829.	5052.	4.6	3967.	4407.	11.1	732.	0.	-100.0
21	39673.	42931.	8.2	4926.	5153.	4.6	4793.	5324.	11.1	807.	0.	-100.0
22	55799.	60381.	8.2	7041.	7366.	4.6	6822.	7579.	11.1	1155.	1283.	11.1
23	62299.	67415.	8.2	8202.	8581.	4.6	7247.	8051.	11.1	1314.	1459.	11.1
	676717.	684141.	1.1	94350.	97909.	3.8	82332.	78328.	- 4.9	14724.	7745.	- 47.4

[a]Without thresholds.
[b]With thresholds.

Q = 868123.

6. CONCLUSIONS

This paper has considered the allocation of resources in a multi-level spatial health care system, where the criterion is to choose locations that satisfy patient's preferences for treatment. To conform more realistically with observed behaviour, thresholds and bounds were introduced as well as parameters reflecting the priorities given to certain services. In the future it is hoped to extend the methods to the equity principle.

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